Yang-Mills Equations

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K. Braun

An alternative QED model based on a physical H(-1/2) Hilbert space enabling a quantum gravity model

The Riemann Hypothesis states that the non-trivial zeros of the Zeta function all have real part one-half. The Hilbert-Polya conjecture states that the imaginary parts of the zeros of the Zeta function corresponds to eigenvalues of an unbounded self-adjoint operator. A (physical) refinement of it is the Berry-Keating conjecture, which is about a (supposed to be Hermitian!) Hamiltonian operator of a particle of mass that is moving under the influence of a potential. Our alternative Zeta function theory framework suggests a H(-1/2) variation representation of the corresponding PDEs.

The classical Yang-Mills theory is a generalization of the Maxwell theory of electromagnetism where the *chromo*-electromagnetic field itself carries charges. For given distributions of electric charges and currents the Maxwell equations determine the corresponding electromagnetic field. The laws by which the currents and charges behave are unknown. The energy tensor for electromagnetic fields is unknown for elementary particles. Matter is built by electromagnetic particles, but the field laws by which they are constituted are unknown, as well. The original inertia law (before Einstein's gravity theory) forced to attribute physical-objective properties to the space-time continuum. Analog to the Maxwell equations (in the framework of a short distance theory) Einstein considered the inertia law as a field property of the space-time continuum.

Based on a H(1/2) energy Hilbert space we propose (analog to the NSE solution) a corresponding (weak) variational Maxwell equation representation. Its corresponding generalization (as described above) leads to a modified QED model. In the same manner as the Serrin gap issue has been resolved (as a result of the reduced regularity requirements) the chromo-electromagnetic field /particles can now carry charges. The open "field law" question above and how "particles" are interacting with each other to exchange energy are modeled in same manner as the coherent/incoherent turbulent flows of its NSE counterpart. The corresponding "zero state energy" model is no longer built on the Hermite polynomials but on its related Hilbert transformed Hermite polynomials, which also span the L(2) Hilbert "test" space.

(KiA) Kirsch A., Hettlich F., The Mathematical Theory of Time-Harmonic Maxwell's Equations, Expansion-, Integral-, and Variational Methods, Springer Cham Heidelberg, New York, Dordrecht, London, 2015,

In the following we collect corresponding relationships to today's differential concept

A Hamiltonian energy field can be (purely) defined by differential form w/o an existing corresponding Lagrange "force" field. This is still possible in case the energy field model is based on regularity assumptions where a Lagrange field is no longer be defined. This concept (i.e. requiring no longer the existence of a "force" field) enables alternative (weak) Yang-Mills and Einstein (energy field) equation representations (in a short distance variation theory Hilbert space framework).

As a classical field theory the Maxwell equations have solutions which travel at the speed of light so that its quantum version should describe massless particles (gluons). However, the postulated phenomenon of color confinement permits only bound states of gluons, forming massive particles. This is the mass gap. Another aspect of confinement is asymptotic freedom which makes it conceivable that quantum Yang-Mills theory exists without restriction to low energy scales. The problem is to establish rigorously the existence of the quantum Yang-Mills theory and a mass gap.

The Maxwell equations (MaJ), the Einstein field equations, the wave equation and the Schrödinger equation can be understood as features of the 4-dimensional Minkowski space (HaT).

The solutions E and H to the Maxwell equations are divergence free and satisfy the Helmholtz equation. Conversely, the divergence free solutions E or H of the Helmholtz equation satisfy the Maxwell equations. Both, the electromagnetic field of a magnetic dipole and the electromagnetic field of an electric dipole satisfy the Silver-Müller radiation conditions. The Cartesian components of any solution to the Maxwell equations satisfying the Silber-Müller radiation conditions also satisfy the Sommerfeld radiation condition for the scalar Helmholtz equation. The converse of this is also true, (CoD) chapter 4.

We note that the natural bilinear (energy) form of the (weak) variation representation of the classical time-harmonic Maxwell equations is not coercive on the whole Sobolev space H(1) (CoM). By adding a certain bilinear form on the boundary of the domain one can make it coercive while changing the natural boundary conditions at the same time. The additional bilinear form contains tangential derivatives of the normal and the tangential components of the field on the boundary, and it vanishes on the subspace of H(1) that consists of fields with either vanishing tangential components or vanishing normal components on the boundary (CoM).

Quantum gravity is a field of theoretical physics that seeks to describe the force of gravity according to the principles of quantum mechanics. Modelling a "*quantum gravitational effect is about defining a consistent scheme in which the space-time metric is treated classically but is coupled to the matter fields which are treated quantum mechanically. .... a deeper theory (still to be found) in which space-time itself was quantized*" (HaS).

We provide a mathematical "*single force"* fermion/boson (particle/energy) quantum element model alternatively to today's 3-force SMEP model with its zoo of different 3-forces depending types of fermions and bosons building on the related Lagrange densities. As a consequence the model covers also the 4th (gravity) nature "force".

The central idea in a nutshell

Plemelj's extended Green formula and mass element concept is based on differential forms, the Stieltjes integral concept in combination with the Pseudodifferential operator (PDO) theory (EsG):

http://www.navier-stokes-equations.com

*Plemelj's extended Green formula enable an inner (energy) product definition of the fractional Sobolev-Hilbert space H(1/2) based on the Poisson equation with **non vanishing** Dirichlet or Neumann boundary conditions.*

We propose to apply Plemelj concepts to build consistent (atomic and subatomic weak H(-a)-variation) Maxwell and Einstein field equation representations in the 4-dimensional Minkowski space-time framework. This leads to variation (Hamiltonian) field representations in a fractional distributional Hilbert space framework. The later one avoids defining a quantum field theory building on the Lagrange formalism and the Dirac function concept. We note that the Lagrange formalism and the Hamiltonian formalism are only equivalent when the Legendre transformation is valid. We further note that the regularity of the Dirac function depends from the space dimension n, whereby our approach requires moderate distributional regularity assumptions (in the size of the Dirac function for space dimension n=1) which are independent from the space dimension. The corresponding extended wave equation (also applied for gravity waves and the formulation of the Huygens principle) as well as the Schrödinger equation can be derived from the Maxwell equations. The extended radiation intensity wave equation defines generalized distortion-free families of progressive spherical waves.

The first rigorous theory of electric field integral equations on screens of arbitrary shape is built on a Hilbert space H(-1/2) as domain for either u, div(u) or u, rot(u):

Smirnow Y. G. The solvability of vector integrodifferential equations for the problem of the diffraction of an electromagnetic field by screens of arbitrary shape

The gasdynamic equations of (plasma) magnetodynamics are very similar to the Navier-Stokes equations of gasdynamics but with interaction terms due to electromagnetic forces (PaS). Of course, these gasdynamic equations should be treated simultaneously with the equations of electromagnetic fields. Those kind of systems are called "Electro-Magneto-Gasdynamics" as it studies the interaction of electromagnetic fields with gasdynamic forces. The case of interaction between a magnetic field and a gasdynamic field is called "Magnetogasdynamics". We propose corresponding variation representation in the above "extended" form as "double layer" "interfacing" model between atomic and subatomic "world".

We claim that the above alternative framework "*solves*" the YME mass gap problem which we see as a consequence of a purely mathematical modelling problem. It is caused by the additively combined three Lagrange "functions"/formalisms (corresponding to three "nature forces") into one "Standard Model of Elementary Particles" (SMEP).

The alternative mathematical solution framework

Today's mathematical tools to model the equations above are vector and tensor analysis, as well as differential forms. Vector analysis cannot be used in the 4-dimensional Minkowski space as there is no exterior product. Tensor analysis gives no physical insight, as there are just tables of coefficients (HaT). The energy tensor for the subatomic area is not known and without a truly infinitesimal geometry (which is different from today's affine geometry) it is impossible to be built.

The spectrum of a selfadjoint operator is real and closed. If the operator is compact then the spectrum is discrete. In case the operator is not compact, but bounded (i.e. continuous) there is a spectral family based representation of the operator built on the Stieltjes integral concept which is only defined for numbers (NeJ). The spectral family of a bounded selfadjoint (Hamiltonian) operator is based on the Hamilton function. The Stieltjes Hamiltonian operator representation enables the definition of related energy and operator norms based on the inner products (Hu,v) and (Hu,Hv).

We propose the following conceptual changes to today's mathematical framework:

1. The L(2) isomorph (quantum state) Hilbert space and the affine connexions/ manifolds concept is replaced by a common fractional (energy) Hilbert space H(1/2) concept, built on a selfadjoint and bounded extended Laplacian operator. The orthogonal sum of the pure point (spectral) subspace H(1) (which is the closed hull of the eigenelements of the Laplacian operator)and its orthogonal complementary space (the closed continuous spectral space) build the fractional (energy) Hilbert space H(1/2).

2. The vector and tensor analysis is replaced by differential forms.

3. The Green formula and the mass density concept is replaced by Plemelj's extended Green formula and his proposed mass element concept. The Dirichlet (integral) energy inner product of the H(1) space is replaced by the inner product of the fractional Hilbert space H(1/2)

4. The Dirac function as an element of the Hilbert space H(-b) with b=n/2-"epsilon" is replaced by the not L(2)-integrable elements of the Hilbert space H(-1/2). As a consequence in case of space dimension n=1 there is a very small regularity loss ("epsilon") prize to be paid, but one gains a large return for all space dimensions n>1.

Some more details

Quantum mechanics theory is about bound states and scattering states of (particles) quanta. One of the key modelling challenges is about how quanta interact leading to concepts like wave package and zero state energy.

Quantum field theory is about energy of atom (particles), radiation fields and coupling of atoms and radiation modelling emission and absorption. One of the key modelling challenges is about the coupling energy without that the atom and the field could not affect each other in any way.

The mathematical framework is about quantum states represented as elements of a separable Hilbert space H with inner product (u,v) which is isomorph to L(2). Modelling energy is enabled by the Hermitian operator, which is assumed to be strong continuous and symmetric with appropriately defined domain. It´s Friedrichs extension leads to a selfadjoint operator with a sub Hilbert space domain H(1) of H(0). The energy value E of the system is then represented by the eigenvalue equation Hu=Eu, resp. the corresponding weak representation E=(Hu,u)/(u,u).

The counterpart of above mentioned challenges is then about when physical phenomena like scattering meet with *infinite* model norm values of the related quantum state u with respect to the H(0)-norm or when required small (but finite) coupling energy values meet with 1/(u,u) infinite model eigenvalue(s).

The proposed modelling idea addresses those challenges by an appropriately built Hilbert space H(-a), a>0, with H(0) as its subspace. This construction (see also (KrR) 4.4, 7.5, 8.4) enables the application of the following

Theorem: The orthogonal complement **W** of a subspace **V** of a Hilbert space **H** is closed. The element u=0 is the only common element of **W** and **V**.

With that in mind we put **V**:=H(0) and **H**:=H(-1/2) and propose **H** as alternative quantum state Hilbert space framework and H(1/2) as its corresponding alternative energy Hilbert space induced by the Hermitian operator with respect to the inner product of **H**. The Hermitian operator is of course still symmetric with domain **V** and the corresponding discrete eigen-pair solutions still exist while the complementary closed space **W** provides the framework to model wave packages (for quantum states with **V**-norm value = zero). The corresponding complementary closed space related to H(1/2) provides the framework to model the coupling energy while the coupling "interaction processes" are acting via the **W-**space channel. The prize to be paid by this modelling approach is that the Lagrange formalism can no longer be applied due to the reduced regularity assumptions as the Legendre transformation is no longer valid.

Referring to (EPN) there can be different starting working titles for the **W**-space like improper, colored, spin vector space, mass gap, coupling & zero-state energy space, spin statistics space, boson states space, particle counts operator domain beyond the Fermi level etc.

J. Plemelj's potential theory ((PlJ) see also (KrR) 8.4 and (CoD) in the context of 3D-scattering theory) enables (weaker) variation representations with respect to the inner (energy) product of H(1/2) applying his extended Green formula and the related derivative operator of the double layer singular integral (PlJ):

let D(u,v) denote the Dirichlet integral (inner product) with domain D and let <u,v> denote the related inner product on the boundary of D. Let further **u** denote the conjugate function of a potential u. For all u,v ex H(1/2) it holds:

i) D(u,u)=<u,d**u**> is positive for all non-constant potential functions u
ii) <v,d**u**>=<.u,d**v**> .

In order to define H(-a) properly we claim that the common RH & NSE solution concepts which are basically about a new "distributional" Hilbert space framework (negative Hilbert scale) in combination with Plemelj's definition of a normal derivative, a mass element (as alternative to today's mass density), a current through a surface and extended Green/Stokes formulas can be applied:

generalized functions on Hilbert spaces in combination with singular integral equations are successfully applied to problems of aerodynamics and electrodynamics (LiL). The corresponding framework is about Hilbert scales with negative scale factor with corresponding Ritz-Galerkin approximation theory for Pseudo-Differential equations. The corresponding energy inner product with its related energy norm corresponds to the extended Green formulas based on J. Plemelj's concept of an alternative normal derivative (PlJ). The related operators with corresponding domains are singular integral operators defined by the single-layer logarithmic potential and the normal derivative of the double layer logarithmic potential (KrR).

The Hilbert scale framework of the NSE solution is built on the eigenpairs of the related self-adjoint Stokes operator. The electromagnetic field is described by the Maxwell equations from which the existence of a scalar potential "phi" and a vector potential **A** can be derived. Those potentials are not unique as there is a transform which keeps the electric and magnetic fields unchanged. This transformation is called 2nd kind gauge transform. Adding this gauge invariance as additional requirement to the potentials (which is basically a divergence-free condition as in the NSE) the Maxwell equations are equivalent to a system of wave equations which is the hyperbolic analogue to the parabolic (heat) equation based on the Laplacian operator.

We propose an analogue building of an appropriate Hilbert scale definition built on the eigenpairs of the self-adjoint Laplace operator with electric and magnetic boundary conditions (WeP) for an alternative purely Maxwell equations based physical model of the today's standard model of particle physics. A "quantized" radiation field can be achieved by same approach as for quantization of the harmonic oscillator. In section "ground state energy" we propose an alternative quantum harmonic oscillator model in a H(-a) framework. Following same concept to the new Maxwell appropriate Hilbert space framework as defined above the strong resp. the H(-a) weak representations and solutions of this model describe the electromagnetic force resp. the electroweak interaction ("force"), building on same model of a "mass element".

The challenge of an appropriate quantum gravity model is about "*defining a consistent scheme in which the space-time metric is treated classically but is coupled to the matter fields which are treated quantum mechanically*" (HaS). Following the concept above this leads to the challenge to define an appropriate Hilbert space replacing the current differentiable manifolds and tensor framework. We note that the manifold assumption to be *differentiable* is a purely mathematical requirement without any physical meaning and necessarity. This reminds somehow to the standard Green formula assumption that the "*grad(V) needs to be defined and finite on the boundary*", which then defines the H(1) regularity requirements to define the Dirichlet (energy inner product) integral.

The energy density of the electromagnetic Maxwell equation field is symmetric and has tensor character. The related momentum density is identical to the energy current. The energy density definitions of the SMEP quantum fields are still in a gray area which results in an indefinite energy tensor T(i,j) of matter to define the Einstein field equations (EiA). This is due to the fact that the matter itself builds the main part of the electromagnetic field.

Based on the Maxwell equations Einstein derived the gravity field equations in analogy to the Newton theory (modelled as Poisson equation) based on the following assumptions/requirements (EiA):

1. the energy density of a gravity model has tensor character T(i,j)

2. the gravity model has to fulfill 4 conservation laws for energy and the linear momentum operator, which is a consequence of the condition div(T(i,j)=0

3. the alternatively to-be-defined differentiation operator to the Laplace operator (Newton theory) should not have higher than two differential quotients of the functions g(i,j) which build the components of the symmetric co-variant fundamental tensor

4. the alternatively to-be-defined differentiation operator to the Laplace operator should be linear with respect to the second differential quotients

5. the divergence of the alternatively to-be-defined differentiation operator vanishes.

From 3 & 4 it follows that this differentiation operator can be built based on the Ricci & Riemann tensor, i.e. has the form R(i,j) + a * g(i,j) * R . From 5 it follows that a=-1/2. Then the analog equation to the (Newton) Poisson equation with a ponderable density constant r on the right side of the Laplacian equation leads to the Einstein field equations, whereby the constant r is replaced by the indefinite energy tensor T(i,j), i.e.

R(i,j) - g(i,j) *R /2 = -c * T(i,j).

In the light of the above idea we suggest a challenge on the assumptions 1-5 above which is basically challenging the strong (classical!) regularity assumptions as baseline to build a quantum gravity model:

Matter has to fulfill the 4 conservation laws for energy and the linear momentum operator. Fulfilling those laws is a consequence of requirements 2. The field equations can be deduced from those assumptions from the Einstein-Hilbert (classical!) action principle representation (ScE).We note that the 4 (classical!) conservation law representations lead to a statement concerning towards a 4-dim. tensor of rank 2 and that a corresponding weak variation representation of the weak field equations would have same energy regularity scale reduction (a=1) as it is the case for the analog classical and weak Poisson equation representations.

Modelling a classical momentum operator leads to requirement 4, while a corresponding weak representation of the momentum operator would require only being linear with respect to the *first* differential quotients.

Plemelj's extended Green formula is a consequence of his proposal to replace the concept of a "mass density" by a "mass element" with corresponding reduced regularity requirements to the potential function. His comment on the (standard) mass density option is the following ((PlJ) §8): "...*such an assumption appears as that fatal restriction that most of the capabilities of the potential is taken away*."

We further note the results of W. Hodge on the potential theory of closed Riemannian manifolds (FlH) 8.4. The work pertains to differential forms alone so one can forget all about vector fields. Building on the Hodge *-operator the space of p-forms on a orientable manifold **M** can be turned into an (infinite dimensional) inner product space. With respect to this inner product for a p-form w and a (p+1)-form v the "delta"-operator applied to v becomes the adjoint operator of the exterior differential operator d applied to w. The Hodge-Laplace operator is defined by d"delta" + "delta"d .

We propose alternative building principles with respect to 1 & 2 in the light of Maxwell's displacement current, Hodge's harmonic (generalized Laplacian) operator, Plemelj's mass element and Werner's selfadjoint extension of the Laplace operator with respect to electric and magnetic boundary conditions to enable a weak representation of the gravity field equations in an appropriately defined energy Hilbert space framework (isomorph to H(1/2) ?).

(HaT): "*The remarkable fact is that the Maxwell equations, Einstein field equations and Schrödinger's equation in quantum mechanics can be understood as features of the 4-dimensional Minkowski space; nothing else is needed. The differential forms are the best technique to describe this fact. Vector analysis cannot be used in 4 dimensions (there is no exterior product) and the tensors give no physical insight (they are just tables of coefficients).*

*Let V be a volume of n dimension which is closed, i.e. there is no boundary, i.e. it holds dV=0. Because space-time is a closed 3-dimensional surface in a 4-dimensional (Minkowski) space this leads to the **principle A**: that the differential of any 3-form in spacetime is zero;* the ***principle B*** is defined according to the *de Rham theorem*".

From the principles A and B consistent Maxwell equations, Einstein equations, curves spaces, wave equation, momentum and forces in the corresponding closed space V can be derived. (HaT) "*Mainly, I think, because in many cases the density of something in a volume V diminishes only as a consequence of there being a stream in time of that something out of that volume, through the surrounding surface dV*."

The chain rule df*a = f*da is valid for all smooth maps f:U--> R(n) and smooth differential forms a in R(n). In (GoV) it is proven that the chain rule is valid also for restricted regularity assumptions: if e.g. f is an element from the (Sobolev) Hilbert space H(1) then the chain rule is valid for any smooth 1-form a on R(n); if additionally, the minors of the Jacobian matrix of f belong to L(2), then the chain rule is valid for any smooth 2-form a on R(n).

The closed 3-dimensional surface space V in the 4-dimensional Minkowski space is proposed as appropriate framework to build the analogue energy Hilbert space H(1/2) for p-forms with appropriate p (GoV). The building principle for the to-be-defined energy Hilbert space is derived from Plemelj's extended Green formula for the Laplacian operator leading to the extended (Dirichlet) energy inner product D(u,u)=<u,d**u**> for u,v ex H(1/2). The baseline framework is a (p-form) Hilbert space built on the inner product as defined in (FlH) §8.4.

Quantum electrodynamics (QED)

QED is the theory of interactions between light (photons) and matter. Photons obey the Bose-Einstein statistic. The state of a set of identical photons is symmetric when those photons are exchanged and its amplitudes are added. A photon can be represented as a solution of the classical appropriately normed Maxwell equations.

Planck postulated that the emission and absorption of radiation in an atom is discontinuous and occurring by quantum leaps which lead him to the definition of the Planck radiation law with correspondingly defined density function. Planck derived his intensity spectral distribution function (modifying statistical mechanics in the style of Boltzmann to an ensemble of photons) by making the assumption that electromagnetic radiation can only be emitted or absorbed in discrete packets, called quanta, of energy. This "solved" the so-called "*ultraviolet catastrophe*" as a consequence of the Rayleigh-Jeans law and the equipartition theorem of classical statistical mechanics for harmonic oscillator modes of a system at equilibrium, where both the power at a given frequency and the total radiated power of a cavity is not observed to be infinite. For large waves and high temperatures the Planck density approximates the Rayleigh-Jeans density, for short waves, low temperatures it approximates the Wien density.

In his famous paper *"Über einen die Erzeugung und Verwandlung des Lichtes betreffenden heuristischen Gesichtspunkt",* Einstein additional postulated that also the electromagnetic radiation itself is built on quanta (photon). This interpretation based on his corresponding statistical analysis of the Planck law was confirmed by the Compton effect. "*The tensor character of the energy density of the electromagnetic field is well known (but only outside of the elementary particles) and the Maxwell equations determine the electric field provided that the distribution of the electric charges and currents are given (*this is about the "what?"). *The laws by which those currents and charges behave (*this is about the "why?" and "how?") *are not known".*

In his famous paper "*The quantum theory of the emission and absorption of radiation*" (1927), Dirac built a quantum theory of radiation which could explain in a unified way interference phenomena and the phenomena of emission and absorption of light by matter. His basic idea was *to treat an atom and the radiation field as a single system whose energy is the sum of three terms: the first one representing the energy of the atom, a second representing the electromagnetic energy of the radiation field, and a small term representing the coupling energy of the atom and the radiation field* (FeE). By this approach he set the foundation to extend the quantization of the electromagnetic field to every classical field. Without the small coupling energy, the atom and the field could not affect each other in any way!!!

While the Einstein paper provides interpretations about the "what", the corresponding "how?" is about the next level of mathematical modelling "how" those "things" happen within an atom. The paper of Dirac was basically the birthday of a quantum field theory which is about quantization of "bosons" fields per to-be-defined force /quanta interaction model per corresponding to-be-defined Lagrange field.

The Lagrange formalism is applied as a standard tool in quantum field theory. Classical quantum field models are quantized requiring/leading to the definition of model (purpose) dependent “quanta elements” which are divided into the two classes, boson fields and fermion “particles”. At the same time the corresponding Hamiltonian formalism, which is equivalent to the Lagrange formalism due to the Legendre transform, is dealing with a model (purpose) independent concept of “energy / action” (see also the quotes from (KaM) below). As a consequence, there is the need to define problem depending boson fields in the quantum world leading to the definition of the 4 “Nature forces” (gravity force, electromagnetic force, weak and strong nuclear forces).

Conceptually this means that there are one-to-one relationships between Lagrange & Hamiltonian formalism and Lagrange field & force definition, and that there is a 1-to-4 relationship between energy & force definitions. As a consequence the today's "Standard Model of Elementary Particles (SMEP)" is about an agglutination of the corresponding 3 gauge (symmetry) groups in the form SU(2)xU(1)xSU(3), whereby U(1) corresponds to the electromagnetic theory, SU(2)xU(1) corresponds to the electroweak theory and SU(3) corresponds to the electro-strong theory. The SMEP group is a Yang-Mills group. Most of the attempts to develop a quantum gravity model more or less try to agglutinate another (gravity) symmetry group (SU(5)xO(10) or larger groups?) to the SMEP group ((KaM) 1.2, " ... *(the) gauge group O(10) is not large enough to accommodate the minimal symmetry of particle physics, namely, SU(3)xSU(2)xU(1); nor can the theory accommodate chiral fermions*"). The prize to be paid for current proposals, are an necessary increase of the common space-time dimension n up to greater than 10.

In the above context we quote from (KaM):

p.4: "*The powerful techniques of renormalization theory developed in quantum field theory over the past decades have failed to eliminate the infinities of quantum gravity…..the problem has been, however, that even the powerful gauge symmetries of Yang-Mills theory and the general covariance of Einstein's equations are insufficient to yield a finite quantum theory of gravity. …..*

p. 10: *This means that general relativity cannot be a renormalizable theory*."

p. 12: "*Because general relativity and quantum mechanics can be derived from a small set of postulates (*continuity, causality, unitarity, locality, point particles*), one or more of these postulates must be wrong. The key must be to drop one or more of these assumptions about Nature on which we have constructed general relativity and quantum mechanics. Over the years several proposals have been made to drop some of our commonsense notions about the universe: continuity, causality, unitarity, locality, point particles*."

p. 18: "*A fundamental theory at Planck energies is also a fundamental theory at ordinary energies."*

In the above context we quote from (GrM):

1.1 The Early Days of dual Models: "*In quantum field theory, the leading nontrivial contributions to the scattering amplitude come from the tree external states, which are particles such as pions that transform in the adjoint representation of the flavor group, which for the three flavor is SU(3) or U(3)."*

1.1.1 The Veneziano Amplitude and Duality: "*... it obeys duality. ... (*in case of high energy behavior of the scattering amplitudes of an elastic scattering process*) it is related to Euler beta function B(u,v) by s- and t-channel amplitudes in the form A(s,t):=B(-a(s),-a(t)) with the Mandelstam variables s, t, u depending from the incoming spin-less particle momenta p(1),p(2) and the outgoing particles of momenta p(3),p(4) (s=-square(p(1)+p(2)), t=-square(p(2)+p(3)), u=-square(p(1)+p(3))"*

7.1 Bosonic Open Strings: *"The understanding of string theory has not yet been developed to the point where one can write down a Lagrangian and follow a standard prescription to deduce rules for constructing Feynman diagrams that provide the loop expansion of the full quantum theory ..."*

7.1.2: "... o*nly physical positive norm states, and not ghosts or other unphysical states, appear at poles ... We wish to show that the physical states that appear at the poles ... have the same property* (i.e. fulfill the Virasoro subsidiary condition).

Our proposed mathematical model replaces the "point particle, locality, continuity" elements by "differentials" (as a model of Leibniz' "living force" monads defined as elements of an appropriately defined Hilbert space enabling generalized Fourier analysis techniques). Observed phenomena like different "force types" or "continuous movements of particles" are described by a single defined "energy quantum (potential)" per related (quantum mechanics or dynamics) mathematical model. The energy interaction ("action") between "quantum elements" is modelled by a "Plemelj double layer potential" (singular (Stieltjes) integral) operator. The energy "transportation/exchange" between quantum elements is modelled by the corresponding normal derivative double layer potential operator. At the same time we postulate that the Lagrange and the Hamiltonian formalism are no longer equivalent, which from a mathematical point of view become a consequence by reduced regularity assumptions in a way that the Legendre transform is no longer valid in the corresponding model. This then goes along with ...

1. ... a corresponding shift on the corresponding Hilbert space domain of the related Hamiltonian operator. As this still keeps a Hilbert space framework the generalized Fourier analysis still can be applied. This means from a physical modelling perspective that the measure room and the observables are infinite accountable, nut not infinite not-accountable (i.e. a continuum in the sense of Cantor with different cardinality)

2. ... the alternative potential, mass element, normal derivative, Green formula concepts of J. Plemelj as described in NSE section.

Our idea (see below) is about an alternative modelling approach, which is basically a building process the other way around, i.e. a kind of "de-quantization/continuity of the continuous & discrete (e.g. continuous & discrete spectrum) to only discrete (countable) spectrum and related eigenfunctions" by orthogonal projection from a distributional Hilbert space H(-a) framework (statuses of identical "mass" elements) and corresponding weak (P)DO equation representation into the subspace H(0). This postulates at the same time that the more "precise/real/object world" physical model is given by the weak PDO equation model (enabling e.g. a "continuous" vacuum energy spectrum), whereby the corresponding classical (approximation) model / solution is an approximation in the standard L(2)-measure room.

With respect to some relationship to the RH solution we mention the alternatively proposed density function to the standard exponential integral function and that the Planck spectrum formula of black body radiation leads to a finite total energy (integral), which is directly related to the value of the Zeta function at point s=4. This means that the Hilbert transform of the Gauss-Weierstrass function provides an alternative Planck spectrum formula which is identical in a weak L(2)-sense to the original formula, but with more appreciated "symmetry/duality" properties.

The point s=4 is directly linked to the space-time dimension in the following way: the number f(l)dl of the free eigenmodes in the volume V related to the wave length interval (l , l+dl) is proportional to the wave length interval dl, i.e. f(l)dl = c*p(l)*Vdl. Since f(l)dl is a dimensionless parameter while Vdl has the dimension of a fourth power of the length therefore p(l) needs to have the reciprocal dimension.

The gravitational (massless graviton) field of spin 2

The Higgs boson (HiP) "explains/describes" the large masses and short ranges of the electroweak interaction "particles" (Z(0), Z(+), Z(-)) on that next level of "granularity". Conceptually the Higgs boson "provides/assigns/enables" mass to the originally massless defined fermions.

The strong interaction ("force") is to "explain"/describe the "existence" of e.g. a "atom kernel", which is enabled by small "glue" particles that hold the quarks together in "conventional" hadrons (e.g. protons and neutrons). Its primary purpose is the describe the binding energy of "atoms" and energy transform for chain reactions, which are triggered by gluons (somehow ...:)).

The quantization approach of classical field is usually based on a Lagrange density, whereby the field per space-time coordinate (considered as dynamic variable) is directly quantized. In order to reduce the degree of freedom from infinite, continuous numbers to an infinite countable numbers (enabling Fourier analysis techniques) one can also consider radiation in a large 3D cube, only, postulating periodic boundary conditions on cube's boundary. The Fourier analysis is then about finding the corresponding "eigenpairs" of the radiation field which are the normal modes (vibrations) with corresponding eigenvalues. This is analogue to the analysis of the harmonic oscillator resp. the vibrating string. The corresponding Hilbert space framework is the test space L(2) = H(0) which is also the standard framework for mathematical probability and statistics theory.

(GrM) 1.1 The Early Days of Dual Models: "*Consistent (renormalizable) quantum field theories seemed to be limited to spin zero, one-half, and one, the known examples being abelian gauge theories and scalar and Yukawa theories. ... although we would include the Yang-Mills theory in the list of consistent theories for spin one."*

(GrM) 1.2 Dual models of everything: "*In general relativity the gravitational field is described by a massless field of spin two, the graviton field. Its nonlinear interactions are determined by a non-abelian local symmetry group, the group of diffeomorphisms of space-time. General relativity was of course part of the inspiration for the invention of the Yang-Mills theory, in which the nonlinearities are determined by an analogous local symmetry group. The Yang-Mills field is a massless field of spin one."*

(GrM) 1.2.1 Duality and the Graviton: "*Quantum gravity has always been a theorist's puzzle par excellence. Experiments offer little guidance except for the bare fact that both quantum mechanics and gravity do play a role in natural law. The characteristic mass scale in quantum gravity is in the Planck mass. ... the real hope for testing quantum gravity has always been that in the course of learning how to make a consistent theory of quantum gravity one might learn how gravity must be unified with other forces*."

Magnetohydrodynamics

(PaS) The study of flow problems of electrically conducting fluids, particularly of ionized gases have been made in the connection with astrophysical and geophysical problems such as sun spot theory, motion of the interstellar gas, origin of earth magnetism, controlled fusion research, plasma jet etc. The most important properties of a plasma from those of an ordinary gas is that in a plasma the electromagnetic forces play a role. Magnetodynamics and plasma dynamics is about the analysis of the flow of plasma and the interaction of the gasdynamic and electromagnetic forces. Plasma dynamics contains Problems from electrical discharge in rarefied gas, propagation of electromagnetic waves in ionized media to the so-called "magnetodynamics".

(GuR): Magnetohydrodynamic is concerned with the motion of electrically conducting fluids in the presence of electric or magnetic fields. The partial differential equations formulated by Lundquist and further discussed by Friedrichs provides to current mathematical model. Friedrichs shows the close parallelism between the theory governing the motion of a compressible fluid whose electrical conductivity may be assumed to be infinite and the theory of conventional gas dynamics. Specifically, the hydrodynamic equations belong to the class of symmetric hyperbolic equations. Because these equations are also reducible, it is possible to develop a theory of shock waves and simple waves in a manner quite similar to that employed in ordinary gas dynamics.

Since magnetohydrodynamics is concerned with the motion of electrically conduction fluids in the presence of electric or magnetic fields, the governing equations are determined by coupling the Maxwell equations for magnetic fields with the usual equations of gas dynamics. The interaction process is complicated since the motion of the fluid generates body forces which modify the motion and induced currents which modify the field. Maxwell’s displacement currents are neglected, and it is assumed that the medium is essential neutral. If it is further assumed that the fluid is ideal, polytropic, inviscid, non-heat conducting compressible medium, whose electrical conductivity is infinite then the phenomenalistic laws can be governed by the Lundquist equations.

(WeH) is concerned with the wave motion in plasma where the plasma is considered as a continuous medium. The Lundquist equations describe such a medium, which represent an isentropic fluid with scalar pressure tensor and infinite conductivity and in which (the sophisticated Maxwell) **displacement currents have been ignored**. It examines only small disturbances imposed on plasma initially with zero flow velocity and constant matter density and magnetic field, i.e. it employs the Lundquist equations linearized about those quantities. The waves are assumed to depend just on the x and y coordinates and the unperturbed magnetic field is taken in the direction of the x-axis. For such wave motion the fundamental solution (extended Green function) is given representing the wave emanating from a point source. By this the Alfven and longitudinal waves are automatically decoupled and many simplifications result.

The Lundquist equations is a well posed mathematical problem. In order that the quantities become physically meaningful the **magnetic field** is assumed to be **divergence free**. This additional assumption introduces complications. This condition is in fact an initial condition rather than a dynamical equation, and the techniques used are not naturally adapted for the imposition of initial conditions. In (WeH) a (extended) Green function solution is provided for the Lundquist equations by **imposing this condition without introducing inconsistencies**. The introduced artifice is basically that instead of assuming delta functions on the right side of the three Lundquist equations there are mass density, magnetic field and motion function representations where the Green function is applied to.

The Green function itself is treated as distribution and determined by inverting its Fourier transform obtained from the PDE.

The fundamental differential equations of the three dimensional unsteady flow of an ideal plasma in magnetogasdynamics belong to the special class of symmetric hyperbolic equations ((PaS), XII). Solutions of this system of differential equations may possess "small" discontinuities only on certain characteristic manifolds in the 3D-space.The possible discontinuities of the quantities magnetic field, the flow velocity, the density and the entropy are governed by relations between those quantities including normal components of the flow velocity and the magnetic field. The characteristic velocity is determined by a related determinately equation derived from those equations. Plemelj's extended Green formula with its related mass element and extended normal component concepts enables a boundary integral representation on those characteristic (shock) manifolds providing physical meanings to the plasma phenomenon of boundary layer flows of magnetogasdynamics.

Relationships between spherical wave solutions of the even n=m+1 space-time radiation wave equation, the Huygens‘ principle & Plemelj's alternative normal derivative definition

The radiation wave equation model is given by the wave equation for a given time dependent radiation intensity (strength) function ((CoR) VI, §5.6,§10.3). Its linkage to the wave equation solution is given by the limit of the radius r to zero for the boundary integral (boundary = surface of the n-ball with radius r with center zero for t>0) over the derivative of the surface normal of u. For even n>2 the spherical waves solution of the radiation wave equation are given up to a space-time dependent constant by the radiation intensity function. The Huygens's principle is valid for the radiation problem based on identical conditions than the initial value problem is valid. Those assumptions are basically about distortion-free families of progressive spherical waves. If those families only exist in case the Huygens's principle is valid and that those families essentially can only exist for n=2 or n=4 is a still open question. In combination with the Hadamard conjecture ((CoR) VI §9.1) this would result into a constitutive classification of the 4-dimensional space-time manifold and the embedded classical Maxwell theory.

Relationships between Leibniz' monads & Plemelj's alternative mass element definition

Physical conception (and the definition of appropriate terms) is about the definition of hypothetical elements, like atoms, forces, mass and electromagnetic fields. Then for example the term "force" becomes the source of measurable physical attributes of a "mass". The primary criterion to decide about "mass" equality is the "momentum", not the "mass" itself.

The mathematical modeling challenge is about the relationship of a mass density and a mass element (particle) (see also the quotes from H. Weyl below). The corresponding physical concept in quantum theory is about fermions and bosons. While fermions represent "real" matter particles fulfilling the Pauli principle to ensure particles' "solidity" and "stability" (i.e. "truly/real" mass elements (e.g. leptons, quarks)) the 4 boson types represent a kind of force field quantization (i.e. a kind of "matter densities") , which define one-to-one the 4 known forces of Nature. Those are the graviton (gravity force), the photon (electric and magnetic force), w-,z-boson (weak nuclear force) and the gluon (strong nuclear force). The physical concept to put fermions and bosons into relationship is about the boson fermion correspondence, which is a canonical transformation (operator) from bosonic Fock space to fermionic Fock space. Some kind of "not symmetry" challenges in this area lead to the mass gap problem of the YME. A famous "solution" to such kind of challenge is the recently "discovered" Higgs boson.

Following Plemelj's definition of a mass element our alternative (purely) mathematical concept somehow "enriches" today's mathematical particle model (which corresponds to a real number) to Leibniz' monad model (which corresponds to a non-standard number) while no longer requiring corresponding "mass densities" (= quantization of related force fields (= boson types)) defined per force type depending defining attributes (velocity, colors, spins). The correspondence "force" or "action" operators are explicitly given by the Stieltjes integral with appropriate integrand in the context of the related physical model. (We note that the "delta" change from a "real number" field to a "non-standard number" field is just about a change from an "ordered" to a "not ordered" field, while the axiom of Archimedes (which ensures a kind of measurability of lengths) is still valid.

The core change

The core change is about moving

from today's zoo of different types of fermions and bosons based on related force models (building on the corresponding Lagrange densities) which

- reflects the today's experimental setup related "quantum/quark" definitions
- already anticipates the matter-field dualism /paradox
- leads to a kind of "self-fulfilling prophecy" interpretations of corresponding observed physical phenomena

to a future single fermion/boson (particle/energy) quantum element Hilbert space concept enabled by Hilbert scale definition building on the Hermitian operator which then is about

- a problem independent bosons vs fermions concept in an appropriately defined distributional Hilbert space framework where "force" becomes a problem dependent phenomenon
- an alternative single mathematical "mass element" concept which is "transcendental" in the sense of E. Kant in the same way as today's real numbers or zoo of all kind of bosons and fermions types
- a still valid measure/test L(2) space as subspace of a distributional Hilbert space to link classical vs. quantum models as corresponding strong vs. weak representations of corresponding (P)DO equations (e.g. Maxwell equations, NSE, harmonic quantum oscillator, Einstein's field equation etc.) enabling to model different "forces" and "same" energy definition
- an alternative "quantization" technique which about the well-known mathematical variation theory putting strong and weak solutions of (P)D equations into relationship
- consistently integrated chemical & quantum potential definitions in one model in sync with a corresponding energy definition this side and beyond the Fermi level. The model is independent from the Dirac function concept with its space dimension depending less regularity requirements.

As a consequence of this new concept

- this is the death of the "color-" and "spin-"full fermions and bosons world, including the even not yet defined additional required attribute(s) list of a graviton (e.g. taste?)
- it provides an (physical) interpretation "*about the inner meaning of the restriction to differential forms (*in the context of required "differentiable" general relativity transformations and related required *differentiable* manifolds*) in order to ensure that both dimensionality and sense derive from the fact that affine geometry holds in the infinitely small. At present it seems to be indispensable since the laws of transformation of most physical quantities are intimately connected with that of the differential dx(i)"* (H. Weyl, "Philosophy of Mathematics and Natural Science" III, 15: "Riemann's point of view, topology", p. 85)

- the integration of Einstein field equations is possible if there is an appropriate initial boundary value model defined which can be treated by variation theory (Hilbert-Einstein functional) to define proper "weak" energy Hilbert space with corresponding weak field equations solution(s).

Some more mathematics

The mathematical concept in quantum mechanics is about differential operators (DO) in combination with appropriate Banach space domains. The famous examples are the location and the momentum operators. The transfer from classical world (described by PDE) into quantum world is basically about weak representations (built on an appropriate energy inner product Variation representation or an operator/graph norm minimization representation) of those PDEs in corresponding reflexive Hilbert space framework. Observables are modeled as selfadjoint operators resp. corresponding orthogonal eigenpairs with discrete eigenvalues.

We propose a generalized Hilbert space framework in combination with the Pseudo-differential operators (PDO) concept, allowing less regulatory assumptions to the underlying Hilbert space domain(s). The transfer to current quantum mechanics models is ensured by orthogonal projection from the generalized Hilbert space domains onto the "standard" L(2) (sub-, test- and measurement- space) Hilbert space with identical discrete eigenpair solutions in L(2), which also still span the L(2) subspace.

The proposed PDO are singular integral operators with corresponding order (e.g. PDO of order +1 replacing the momentum operator; PDO of order +0 replacing the location operator, etc.), while (newly) based on a common domain. The integral operators are extended to the Stieltjes integral concept, which links PDO with the concept of differentials. In this sense differentials become now "real" physical objects (which also fits to Plemelj's concept of a mass element). The (rotation invariant) Riesz operators are of order +0. By "partial integration" Riesz operators applied to differentials (in the sense of a Stieltjes integral) lead to Calderon-Zygmund operators of order +1. This equivalence can be interpreted as "located, rotating differentials" are identical with a corresponding "momentum energy".

Finally, the above results into a truly infinitesimal geometry (Hilbert space equipped with an inner product) replacing today's Weyl's affine connexion manifolds framework (equipped with an exterior derivative product, only), overcoming the still open (Lie-) "contacting body" problem, when e.g. a photon meets a photon.

Relationships to the solutions of RH & NSE problems

From Weyl H., "*Philosophy of Mathematics and Natural Science*", we quote:

p. 179: …"*Newton's gravity appears as an instantaneous action into distance. When only nearby action is considered admissible, "ether" theories of gravity arise. Based on the idea of a field for the electric phenomena Maxwell found that the field propagates from the centers of excitation not instantaneously but with the velocity of light. Nearby action laws, in the form of differential equations, connect the physical quantities characteristics of matter and fields, namely charge and current densities and electrical and magnetic field strengths. … the field transmits momentum as well as energy from one body to another; a radiating body not only loses energy, but as it radiates light in one direction it recoils in the opposite direction. In the field we therefore have spatially localized energy and momentum. The scalar densities and the components of the vectorial current densities of energy and momentum can be computed by means of simple laws from the two field strengths. ...*

p. 165. .." *But in a completely homogeneous substance without any quality, the recognization of the same place is as impossible as that of the same point in homogeneous space. For this reason Democritus's idea necessarily leads to atomism and to the recognization of empty space. It is also their atomic constitution that explains the different densitiy of bodies, their capacity of rarefaction and condensation - namely by a mixtures of atoms and empty space in changing proportions of volume*. ..."

We claim that the NSE solution concept can also be applied to the Maxwell equations overcoming current electric & magnetic initial-boundary value problems (regularity assumptions (e.g. to define *densities*) and boundary layer interaction (normal derivative of e.g. double layer potential) challenges).The Hilbert scale concept in combination with appropriately defined self-adjoint PDO and extended Green formulas provides an alternative framework to address the above mentioned interaction challenges of transmitted momentum of fields and interaction with related bodies:

specifically:

- the application of the proposed NSE solution to Maxwell equations (and related references) is sketched in the corresponding NSE paper

- the solution concept for the NSE open question is about rotation-invariant fluids (as the Riesz operators are rotation invariant) as elements of a Hilbert space with negative (distributional) Hilbert scale defined by the eigenpairs of the Stokes operator. It is enabled by the alternative "normal" derivative definition provided by J. Plemelj (which requires reduced regularity assumptions to the solution u in the size of a reduction from C(1) to C(0)), leading to a energy Hilbert space H(1/2), which does not create a Serrin gap for n=3 and H(1/2) energy Hilbert space

- the proposed solution concept for the Yang-Mills equations and the mass gap is about rotation-invariant differentials as elements of a Hilbert space domain of Riesz operators with negative (distributional) Hilbert scale factor defined by the eigenpairs of the Hamiltonian operator.

- the proposed solution concept for an alternative ground state energy model is about rotation-invariant differentials as elements of a Hilbert space with negative (distributional) Hilbert scale defined by the eigenpairs of the Hamiltonian operator.

We claim that the provided NSE solution, which is addressing the too strong/restrictive mathematical model assumption of the NSE, can be also applied to the Maxwell equations, as well as to the zero state energy "problem" of the harmonic quantum oscillator. The d'Alembert paradox (fluid vs. velocity/momentum) is related to those too restrictive assumptions for the NSE, while the Schrödinger paradox (particle vs. wave/momentum) is related to the latter one.

We note that

- the Biot-Savart law, which provides the foundation to derive the Maxwell equations, has been derived empirically, only

- the Biot-Savart singular integral operator is a PDO of order -1

- the Biot-Savart formula enables the transformation between the velocity and the vorticity representation of the NSE

- the Hilbert transform plays a key role in the Constantin-Lax-Majda (CLM) one dimensional model for the 3D-Navier-Stokes vorticity equation.

We further note that there is a relationship between the confluent hypergeometric function (as applied in our RH solution) and an integral representation of the NSE, derived by Fourier coefficient analysis (Peralta-Fabi R., An Integral Representation of the Navier-Stokes Equations-I, Revista Mexicana de Física 31 No. 1 (1984) 57-67).

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